Smoothing Mixed Traffic with Robust Data-driven Predictive Control for Connected Autonomous Vehicles[1]

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2. Review of Data-Enabled Predictive Control (DeePC)
3. Robust DeePC in Smoothing Mixed Traffic
4. Conclusion and Future Work

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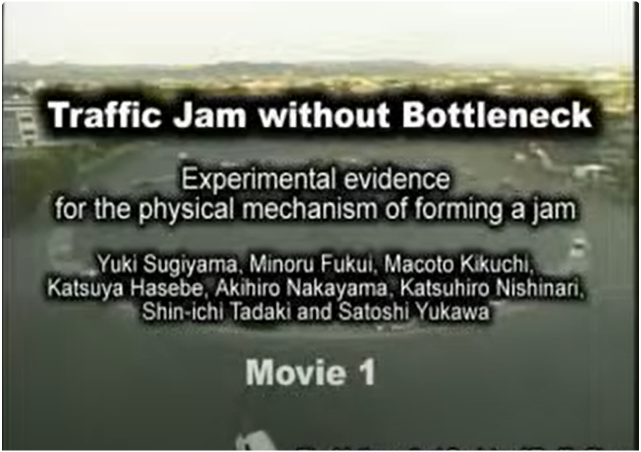
Stop-and-go Traffic Waves

Small perturbations of vehicle motion may progate into large periodic speed fluctuations, which (1) lowers traffic efficiency and (2) reduces driving safety.



https://www.youtube.com/watch?v=7wm-pZp_mi0

Traffic Wave Mitigation

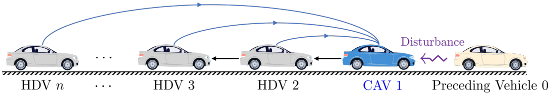
Experimental Validation of Data-EnablEd Predictive Leading Cruise Control (DeeP-LCC) in Dissipating Traffic Waves

https://www.youtube.com/watch?v=ZZ2cWhapqpc

Mixed Traffic System

System Setup:

- The mixed traffic system consists of one connected and autonomous vehicle (CAV) and $n-1$ human-driven vehicles (HDV). All these vehicles follow a head vehicle.
- For the i-th vehicle, its position, velocity and acceleration are denoted as p_i, v_i and a_i. The spacing of vehicle i is $s_i = p_{i-1} - p_i$.



Vehicle Dynamics:

- HDV: $a_i(t) = v_i(t) = F_i(s_i(t), \dot{s}_i(t), v_i(t))$.
- CAV: Control input $a_i(t) = u_i(t)$.

Mixed Traffic System

Combine all dynamics of HDVs and CAVs after linearization, the system model of mixed traffic is

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + H\epsilon(k), \\ \quad\quad y(k) = Cx(k), \end{cases} \tag{1}$$

where we define

$$x(t) = [\tilde{s}_1(t), \tilde{v}_1(t), \tilde{s}_2(t), \tilde{v}_2(t), \dots, \tilde{s}_n(t), \tilde{v}_n(t)]^{\mathsf{T}} \in \mathbb{R}^{2n},$$
$$y(t) = [\tilde{v}_1(t), \tilde{v}_2(t), \dots, \tilde{v}_n(t), \tilde{s}_1(t)]^{\mathsf{T}} \in \mathbb{R}^{n+1}.$$

The input is the acceleration of the CAV and the disturbance is the velocity error of the preceding vehicle.

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The input is the acceleration of the CAV and the disturbance is the velocity error of the preceding vehicle.

Goal: Design control input for unknown system (1) with available traffic data.

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DeePC Overview

Consider the well-known receding horizon predictive control problem with unknown system model and initial state

$$\min_{x,u,y} \sum_{k=t}^{t+N-1} \left(\|y(k)\|_Q^2 + \|u(k)\|_R^2 \right)$$

$$\text{subject to} \quad x(k+1) = Ax(k) + Bu(k), \tag{2a}$$

$$y(k) = Cx(k) + Du(k), \tag{2b}$$

$$x(t) = x_{\text{ini}},$$

$$u(k) \in \mathcal{U}, \; y(k) \in \mathcal{Y},$$

and we only have access to

- Input/output trajectory of length-T (Offline data).
- The most recent past input/output sequence of length-T_{ini} (Online data).

Data-Eanbled Predictive Control (Coulson, 2019)**:**

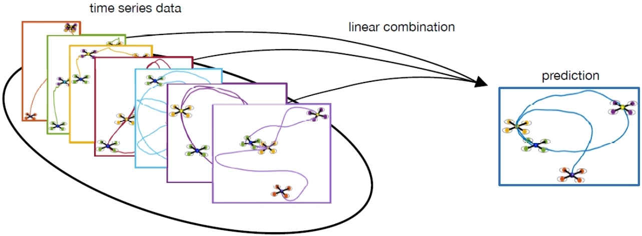
1. Construct a data-driven representation for system (2).
2. Ensure the predicted trajectory satisfy the initial condition of the system.

DeePC Overview

Lemma 1 (Fundamental Lemma)

Suppose that system (2) is controllable. Given a length-T I/O trajectory: $u_d \in \mathbb{R}^{mT}$, $y_d \in \mathbb{R}^{pT}$ and assume these data is rich enough, then a length-L I/O sequence (u_s, y_s) is a valid trajectory of (2) if and only if there exists a $g \in \mathbb{R}^{T-L+1}$ such that

$$\begin{bmatrix} \mathcal{H}_L(u_d) \\ \mathcal{H}_L(y_d) \end{bmatrix} g = \begin{bmatrix} u_s \\ y_s \end{bmatrix}.$$



(Markovsky et al., 2023)

DeePC Formulation

Model Predictive Control:

$$\min_{x,u,y} \sum_{k=t}^{t+N-1} \left(\|y(k) - y_{\mathrm{r}}(k)\|_Q^2 + \|u(k)\|_R^2\right)$$

$$\text{subject to} \quad x(k+1) = \boxed{A}\,x(k) + \boxed{B}\,u(k), \quad k \in [t, t+N-1]$$

$$y(k) = \boxed{C}\,x(k) + \boxed{D}\,u(k), \quad k \in [t, t+N-1]$$

$$x(t) = \boxed{x_{\text{ini}}},$$

$$u(k) \in \mathcal{U},\ y(k) \in \mathcal{Y}, \quad k \in [t, t+N-1],$$

Data-enabled Predictive Control:

$$\min_{g,u,y} \sum_{k=t}^{t+N-1} \left(\|y(k)\|_Q^2 + \|u(k)\|_R^2\right)$$

$$\text{subject to} \quad \boxed{\begin{bmatrix} U_{\mathrm{P}} \\ Y_{\mathrm{P}} \\ U_{\mathrm{F}} \\ Y_{\mathrm{F}} \end{bmatrix}} g = \begin{bmatrix} \boxed{u_{\text{ini}}} \\ \boxed{y_{\text{ini}}} \\ u \\ y \end{bmatrix}$$

$$u \in \mathcal{U}, y \in \mathcal{Y}$$

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DeePC with Disturbance Estimation

We can treat the disturbance as another input and form the system as

$$\begin{cases} x(k+1) = Ax(k) + [B \quad H] \begin{bmatrix} u(k) \\ \epsilon(k) \end{bmatrix} = Ax(k) + \hat{B}\hat{u}(k), \\ y(k) = Cx(k), \end{cases}$$

The form of the DeePC becomes

$$\begin{aligned} \min_{g,\sigma_y,u,\epsilon,y} \quad & V(u,y) + \lambda_g ||g||_2^2 + \lambda_y ||\sigma_y||_2^2 \\ \text{subject to} \quad & \begin{bmatrix} U_\mathrm{P} \\ E_\mathrm{P} \\ Y_\mathrm{P} \\ U_\mathrm{F} \\ E_\mathrm{F} \\ Y_\mathrm{F} \end{bmatrix} g = \begin{bmatrix} u_\mathrm{ini} \\ \epsilon_\mathrm{ini} \\ y_\mathrm{ini} + \sigma_y \\ u \\ \epsilon \\ y \end{bmatrix}, \\ & \tilde{s}_\mathrm{min} \leq G_1 y \leq \tilde{s}_\mathrm{max}, \\ & u_\mathrm{min} \leq u \leq u_\mathrm{max}, \\ & \epsilon = \epsilon_\mathrm{est}. \end{aligned}$$

Robust DeePC

We can estimate a potential disturbance set to robustify the problem

$$
\begin{aligned}
\min_{g,\sigma_y,u,\epsilon,y} \; \max_{\epsilon \in \mathcal{W}} \quad & V(u,y) + \lambda_g ||g||_2^2 + \lambda_y ||\sigma_y||_2^2 \\
\text{subject to} \quad & \begin{bmatrix} U_P \\ E_P \\ Y_P \\ U_F \\ E_F \\ Y_F \end{bmatrix} g = \begin{bmatrix} u_{\text{ini}} \\ \epsilon_{\text{ini}} \\ y_{\text{ini}} + \sigma_y \\ u \\ \epsilon \\ y \end{bmatrix}, \\
& \tilde{s}_{\min} \leq G_1 y \leq \tilde{s}_{\max}, \quad \forall \epsilon \in \mathcal{W}, \\
& u_{\min} \leq u \leq u_{\max}.
\end{aligned}
$$

$$
\begin{aligned}
\min_{x,t} \quad & t \\
\text{subject to} \quad & x^{\mathsf{T}} M x + d^{\mathsf{T}} x \leq t, \quad \forall \epsilon \in \mathcal{W}, \\
& \tilde{s}_{\min} \leq P_1 x + c_1 \leq \tilde{s}_{\max}, \quad \forall \epsilon \in \mathcal{W}, \\
& u_{\min} \leq P_2 x \leq u_{\max}.
\end{aligned}
$$

where $x = \text{col}(u, \sigma_y, \epsilon)$ is decision variable.

Robust DeePC

We can estimate a potential disturbance set to robustify the problem

$$\min_{g,\sigma_y,u,\epsilon,y} \ \max_{\epsilon\in\mathcal{W}} \quad V(u,y)+\lambda_g||g||_2^2+\lambda_y||\sigma_y||_2^2$$

$$\text{subject to} \quad \begin{bmatrix} U_P \\ E_P \\ Y_P \\ U_F \\ E_F \\ Y_F \end{bmatrix} g = \begin{bmatrix} u_{ini} \\ \epsilon_{ini} \\ y_{ini}+\sigma_y \\ u \\ \epsilon \\ y \end{bmatrix},$$

$$\tilde{s}_{min} \le G_1 y \le \tilde{s}_{max}, \quad \forall \epsilon \in \mathcal{W},$$

$$u_{min} \le u \le u_{max}.$$

$$\min_{x,t} \quad t$$

$$\text{subject to} \quad x^{\mathsf{T}} M x + d^{\mathsf{T}} x \le t, \quad \forall \epsilon \in \mathcal{W},$$

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Advantages:

- Increase the safety guarantee.
- Decrease the required amount of offline data.

Trade-off:

- Increase the computational cost.
- An accurate disturbance estimation method is needed.

Disturbance Estimation

The estimated disturbance set is modeled as an N-dimensional polytope

$$\mathcal{W} = \{\epsilon \in \mathbb{R}^N | A_\epsilon \epsilon \leq b_\epsilon\},$$

where $A_\epsilon = [I; -I]$, $b_\epsilon = [\epsilon_{\text{max}}; -\epsilon_{\text{min}}]$.

Estimation Methods:

- Constant bound: the **disturbance** variation for the future disturbance trajectory is close to its past trajectory.
- Time-varying bound: the variation of the **acceleration** for the future disturbance trajectory is close to its past trajectory.

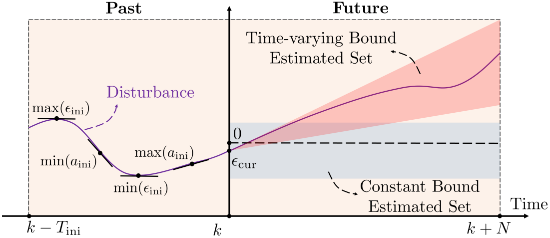

Figure: Schematic of disturbance estimation methods

Efficient Computations

Method I: Vertex-based. The compact polytope $\mathcal{W}$ can be represented as the convex hull of its extreme points $\mathcal{W} = \mathrm{conv}(\omega_1, \ldots, \omega_{n_\mathrm{v}})$ where $n_\mathrm{v} = 2^N$.

$$\begin{aligned}
\min_{x,t} \quad & t \\
\text{subject to} \quad & x_j^\mathsf{T} M x_j + d^\mathsf{T} x_j \leq t,\ j = 1, \ldots, n_\mathrm{v}, && \text{(3a)} \\
& \tilde{s}_{\min} \leq P_1 x_j + c_1 \leq \tilde{s}_{\max},\ j = 1, \ldots, n_\mathrm{v}, && \text{(3b)} \\
& u_{\min} \leq P_2 x \leq u_{\max}. && \text{(3c)}
\end{aligned}$$

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& \tilde{s}_{\min} \leq P_1 x_j + c_1 \leq \tilde{s}_{\max},\ j = 1, \ldots, n_\mathrm{v}, && \text{(3b)} \\
& u_{\min} \leq P_2 x \leq u_{\max}. && \text{(3c)}
\end{aligned}
$$

Method II: Duality-based. Change the affine constraint into its dual problem and form the problem as a min-min problem.

$$
\begin{aligned}
\min_{x_\mathrm{d}, t, \lambda_1, \lambda_2} \quad & t \\
\text{subject to} \quad & p_{l,\mathrm{d}}^\mathsf{T} x_\mathrm{d} + b_\epsilon^\mathsf{T} \lambda_{l,1} + c_{1,l} \leq \tilde{s}_{\max}, && \text{(4a)} \\
& A_\epsilon^\mathsf{T} \lambda_{l,1} - p_{l,\epsilon} = 0, && \text{(4b)} \\
& -p_{l,\mathrm{d}}^\mathsf{T} x_\mathrm{d} + b_\epsilon^\mathsf{T} \lambda_{l,2} - c_{1,l} \leq -\tilde{s}_{\min}, && \text{(4c)} \\
& A_\epsilon^\mathsf{T} \lambda_{l,2} + p_{l,\epsilon} = 0, && \text{(4d)} \\
& \lambda_{l,1} \geq 0, \lambda_{l,2} \geq 0,\ l = 1, 2, \ldots, N, && \text{(4e)} \\
& \text{(3a), (3c).}
\end{aligned}
$$

Complexity with down-sampling strategy

Down-sampling Strategy: Choose one point for every T_s steps to approximate the disturbance trajectory and perform linear interpolation.

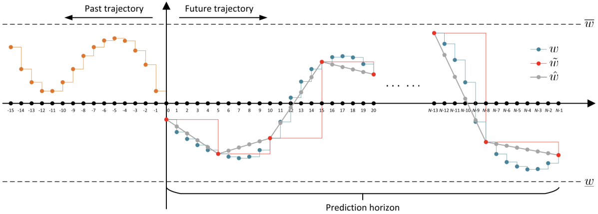

Figure: Illustration of down-sampling strategy (Huang et al., 2023)

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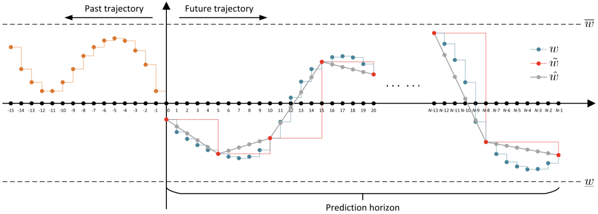

Figure: Illustration of down-sampling strategy (Huang et al., 2023)

Table: Complexity comparison between Method I and Method II.

	Decision Variables Number	Constraints Number
M1	$(n+1)T_{\mathrm{ini}} + N + 1$	$2^N + N \cdot 2^{N+1} + 2N$
M2	$(n+1)T_{\mathrm{ini}} + N + 1 + 4N^2$	$2^N + 2N(3N+2)$
M1 (L)	$(n+1)T_{\mathrm{ini}} + N + 1$	$2^{n_\epsilon} + N \cdot 2^{n_\epsilon+1} + 2N$
M2 (L)	$(n+1)T_{\mathrm{ini}} + N + 1 + 4Nn_\epsilon$	$2^{n_\epsilon} + 2N(3n_\epsilon + 2)$

Experiment Setup

System Setup:

- We consider the CAV 1 is followed by 4 HDVs, and there are three vehicles in front of the head vehicle 0.

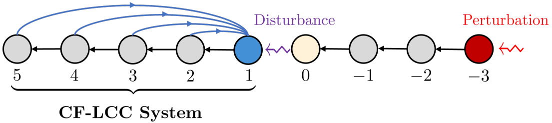

Figure: Simulation scenario

- The length of pre-collected data sets are $T = 500$ for a small data set and $T = 1500$ for a large data set.

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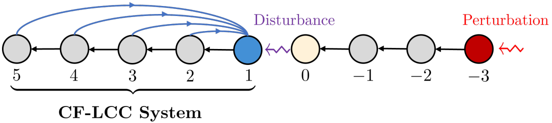


Figure: Simulation scenario

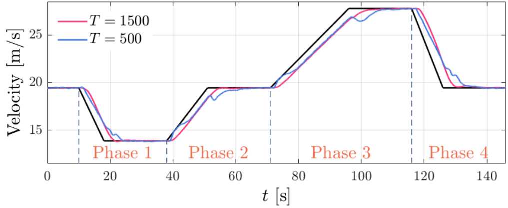
- The length of pre-collected data sets are $T = 500$ for a small data set and $T = 1500$ for a large data set.

Scenarios:

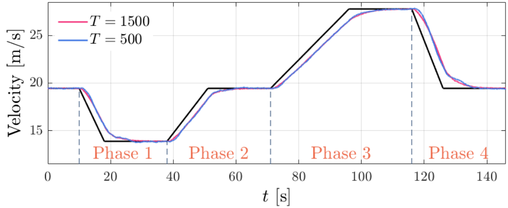
- Comprehensive simulation: design a velocity profile of the leading vehicle and check the tracking performance of the controller.
- Braking scenario: the leading vehicle will suddenly brake with the maximum deceleration to validate the safety performance of the controller

Comprehensive Experiment

We first validate the control performance of robust DeeP-LCC in a comprehensive scenario.



(a) DeeP-LCC



(b) Robust DeeP-LCC

Comprehensive Experiment

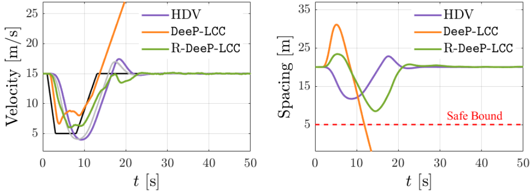
The robust DeeP-LCC decreases the fuel consumption especially at the braking phase.

Table: Fuel Consumption in Comprehensive Experiment (unit: mL)

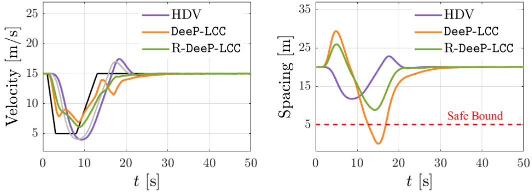
	All HDVs	DeeP-LCC	Robust DeeP-LCC
Phase 1	145.59	141.02 (↓ 3.14%)	135.60 (↓ **6.86%**)
Phase 2	314.77	312.95 (↓ 0.58%)	311.83 (↓ **0.94%**)
Phase 3	725.28	723.95 (↓ 0.18%)	722.88 (↓ **0.33%**)
Phase 4	259.05	246.16 (↓ 4.97%)	237.89 (↓ **8.17%**)
Total Process	1530.15	1509.6 (↓ 1.54%)	1493.6(↓ **2.39%**)

Braking Scenario

We then validate the safety performance of robust DeeP-LCC in the braking scenario.



(c) Small offline data set with $T = 500$



(d) Large offline data set with $T = 1500$

Braking Scenario

The robust DeeP-LCC provides better safety guarantee.

Table: Collision and Safety Constraint violation rate

	DeeP-LCC		Robust DeeP-LCC	
	$T = 500$	$T = 1500$	$T = 500$	$T = 1500$
Violation Rate	74%	62%	**5%**	**0%**
Emergency Rate	66%	51%	**4%**	**0%**

Violation: the CAV's spacing deviates more than $1\ \mathrm{m}$ from safety range.
Emergency: the CAV's spacing deviates over $5\ \mathrm{m}$ from safety range

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Conclusion: The robust formulation with relative accurate disturbance set estimation methods

- Provide a stronger safety guarantee.
- Improve the control performance.
- Allow for the applicability of a smaller data set.

Future work:

- Learning-based estimation for future disturbances.
- Incorporation of communication-delayed traffic data.

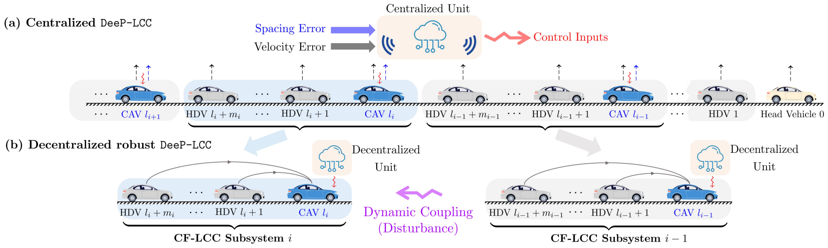

Figure: Schematic of centralized and decentralized robust DeeP-LCC

Acknowledgements

Thank you!

Q&A

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